Imperial College
London

4 – Intensity Transformation

Prof Peter YK Cheung

Dyson School of Design Engineering

URL: www.ee.ic.ac.uk/pcheung/teaching/DE4_DVS/
E-mail: p.cheung@imperial.ac.uk

Spatial Domain Processing

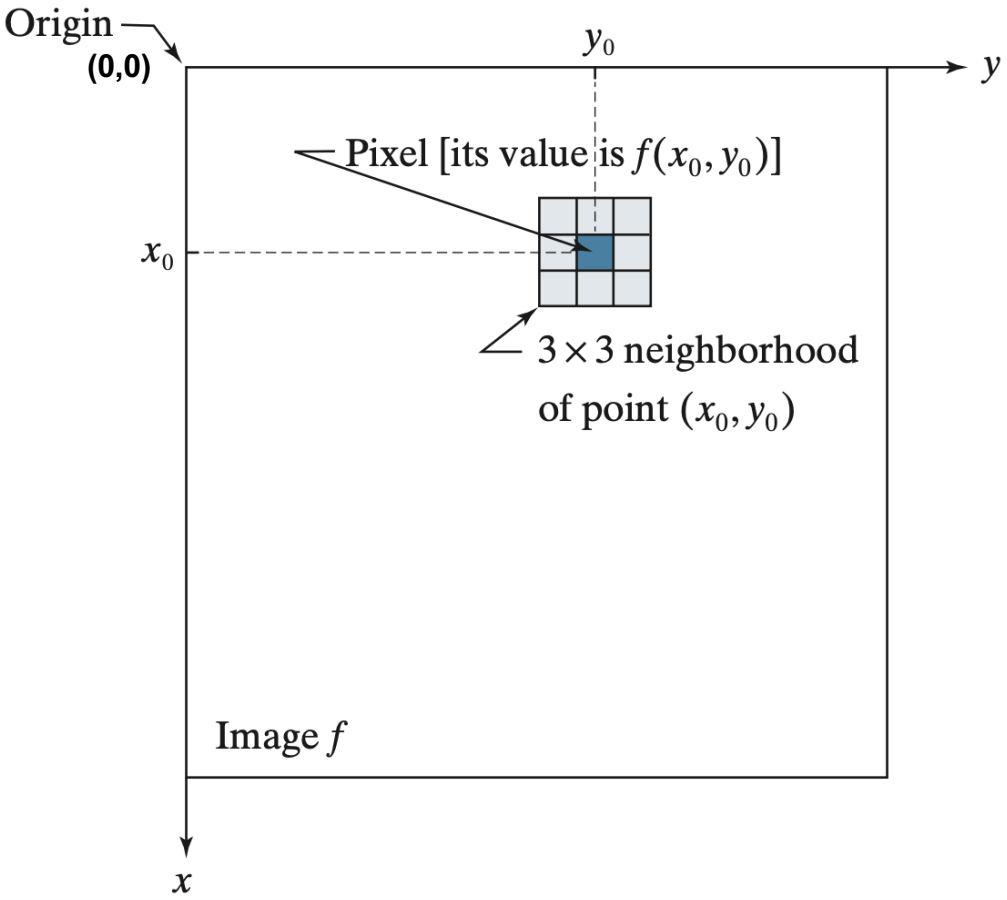
- Visual data is captured and stored in computer
- Only using intensity (or luminous) information
- Scene is sampled at regular spatial grid in X and Y direction
- General equation of processing is:

$$g(x,y) = T\left[f(x,y)\right]$$

  - $f(x,y)$ is the captured visual information at (x,y)
  - $T[.]$ is the operator defined over a neighborhood of point (x, y)
  - $g(x,y)$ is the output image at (x,y)

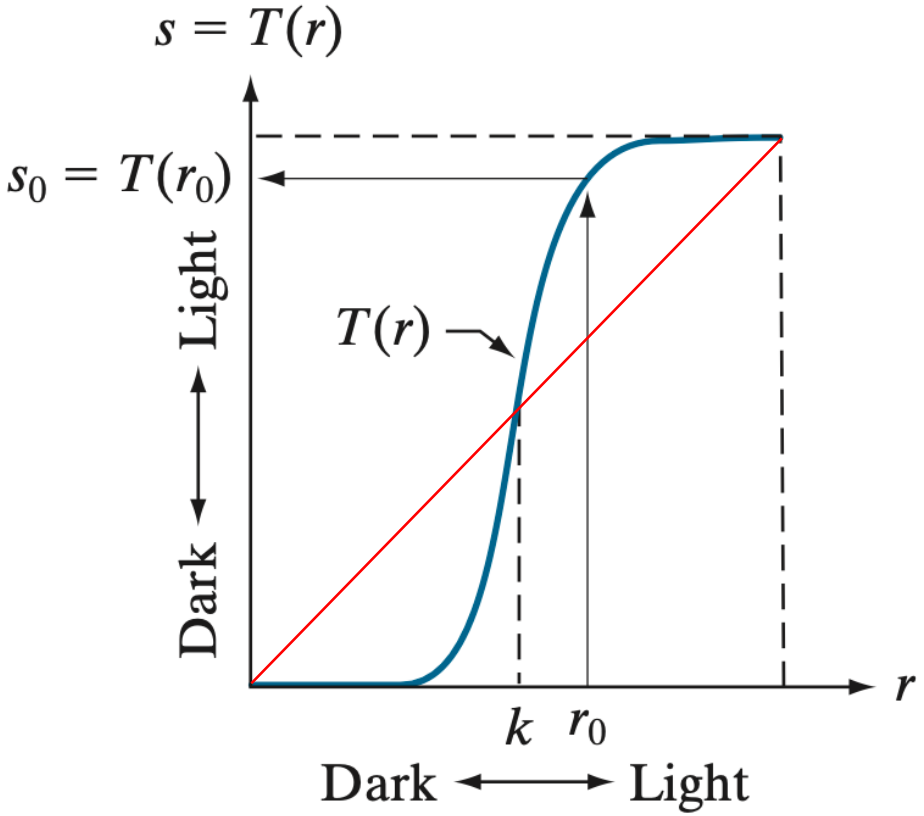
Neighbourhood operation

Matlab origin – (1,1)

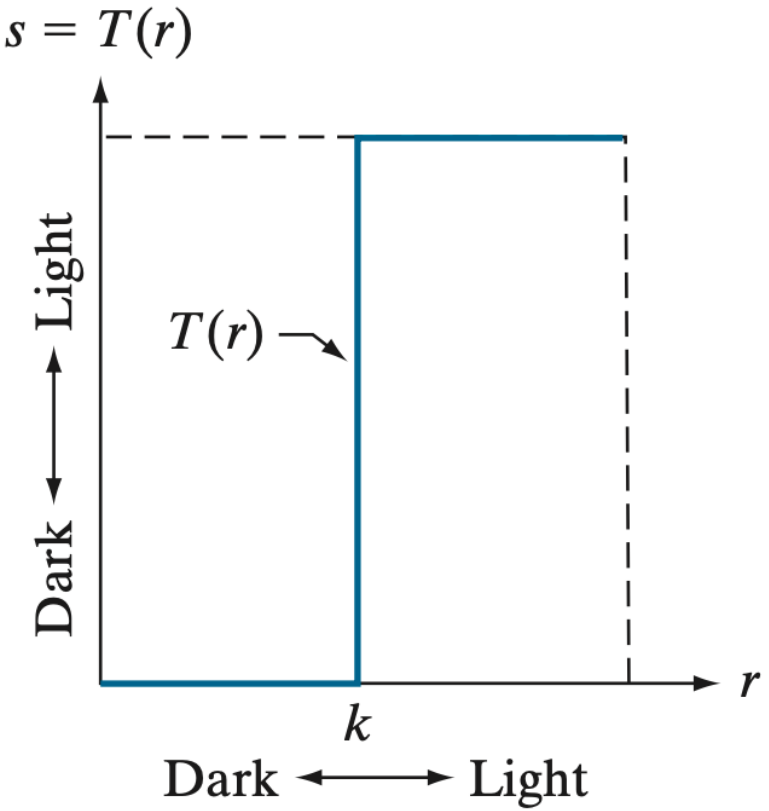


- (x_0, y_0) – arbitrary location in image
- Neighbourhood is a rectangle centred on (x_0, y_0)
- This shows a 3x3 neighbourhood
- Operator T is applied to the pixel values of neighbouring pixel surrounding $f(x_0, y_0)$
- The result of T is output image value $g(x_0, y_0)$
- Processing the entire image requires such an operation performed over the entire image, pixel-by-pixel, starting from the origin

Examples of Operator

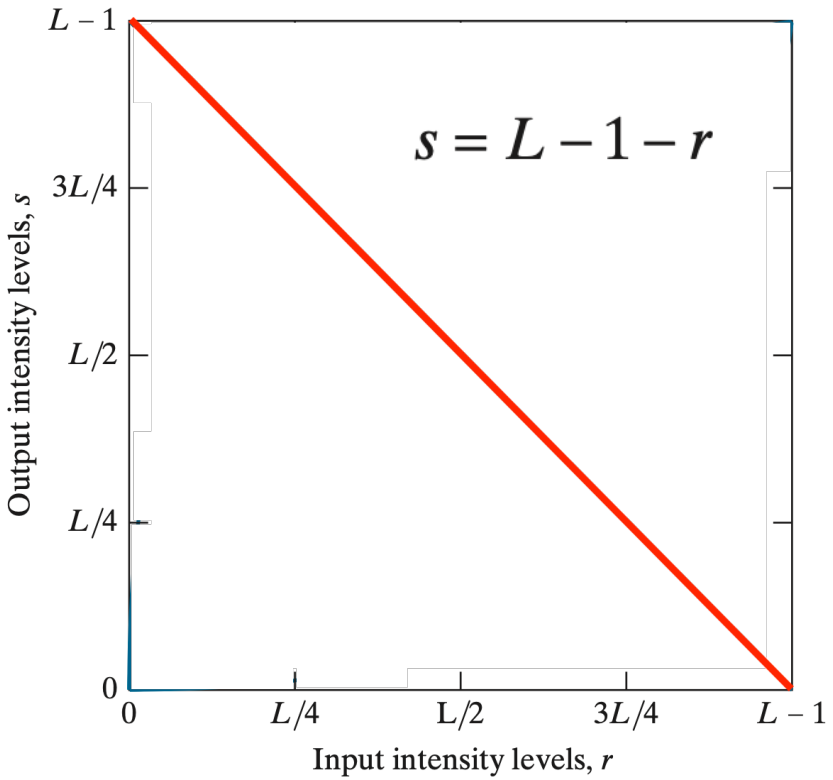


Contrast Stretching function

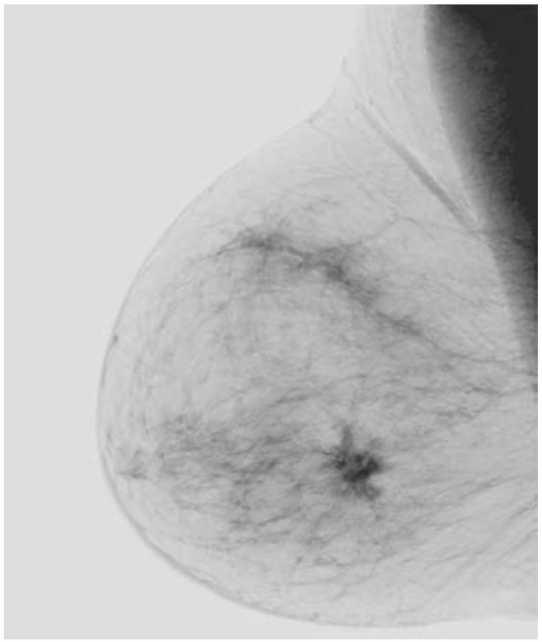


Thresholding function

Negative Transformation



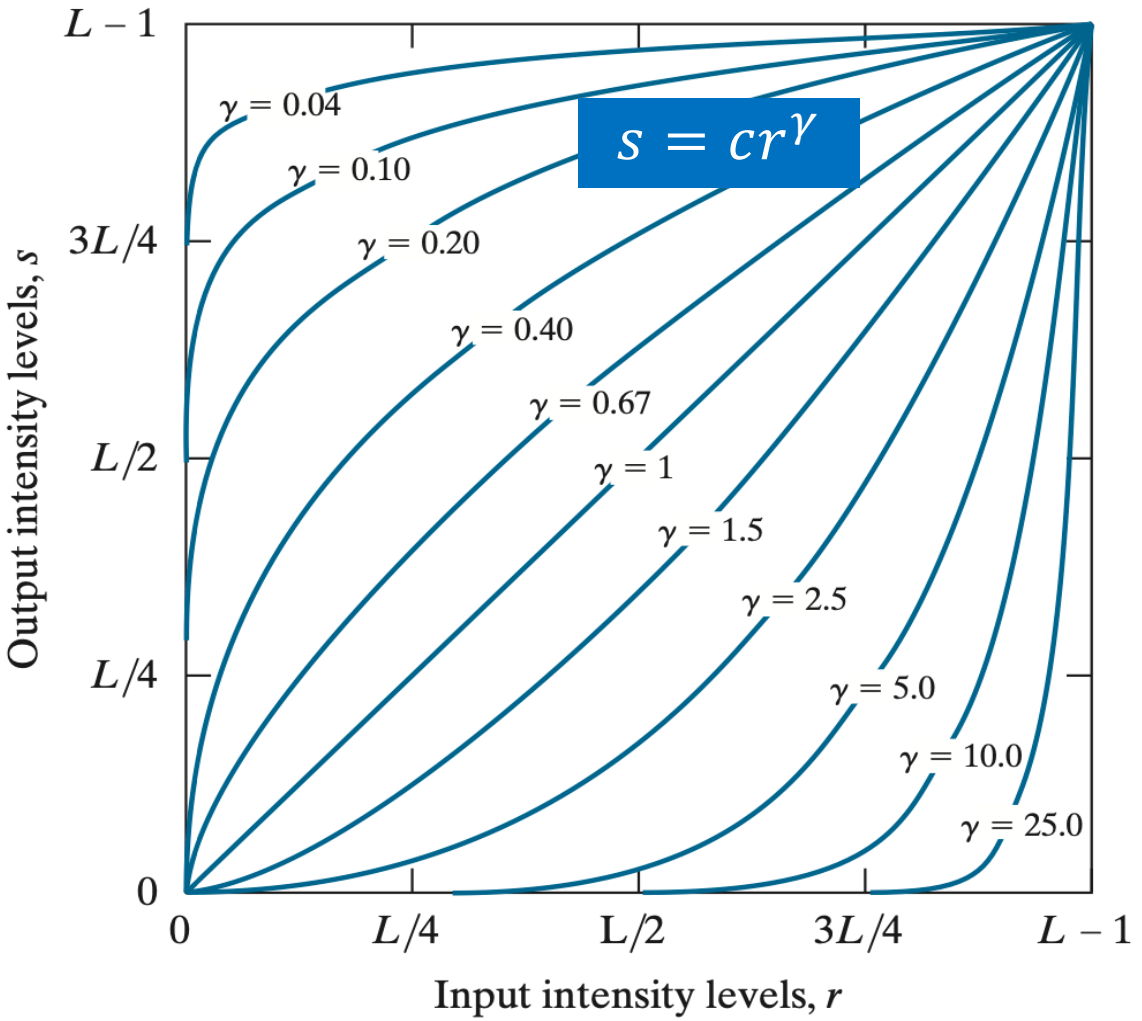
Original Image



Negative Image

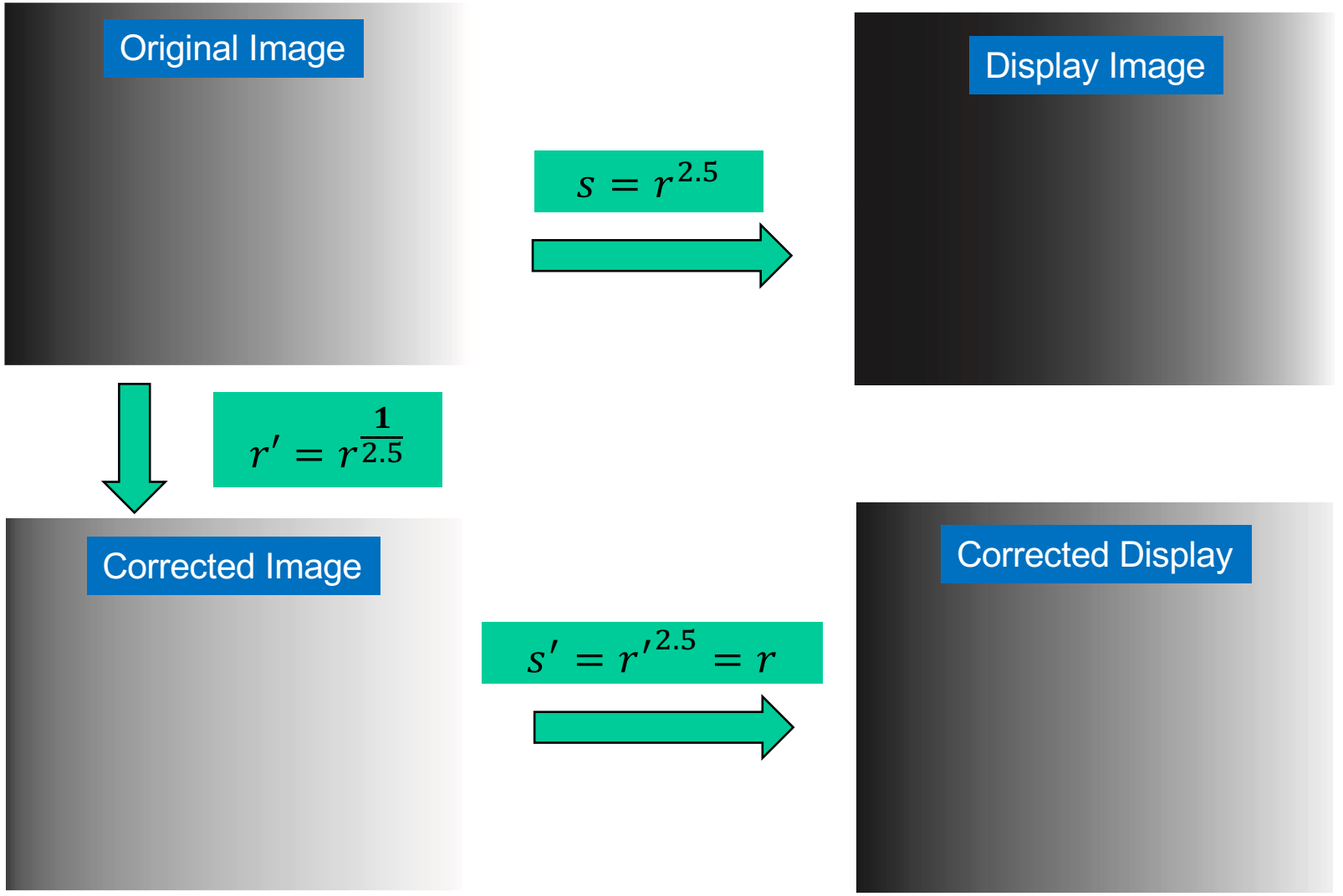
- r is the original pixel value in range [0, L-1]
- s is the resulting pixel value after negative operator

Power-law Transformation (Gamma Correction)



- c and γ are positive constants
- Better known as Gamma Correction
- Assume c = 1
- Used for processing an image for display on monitors
- Different values of γ produce a family of functions
- $\gamma = 1$ means image is not changed
- $\gamma > 1$ produces in darker image
- $\gamma < 1$ produces a ligher image
- $s = r^{\gamma}$ and $s = r^{\frac{1}{\gamma}}$ are inverse of each other

Gamma Correction for Monitor Display



Power-law Contrast Enhancement (1)

$s = r^{\gamma}$

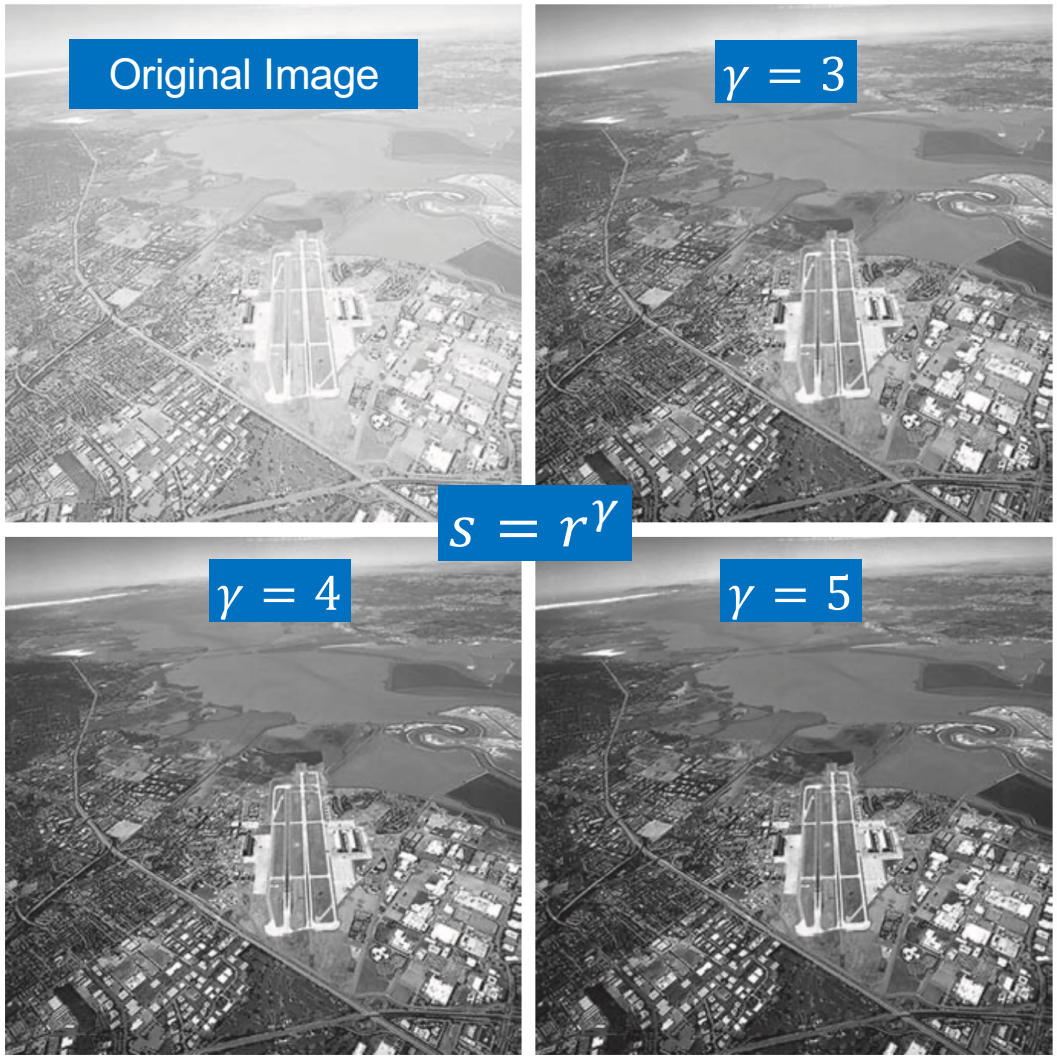
- $\gamma < 1$ lightens the image and shows break in spine

Original Image

$\gamma = 0.6$

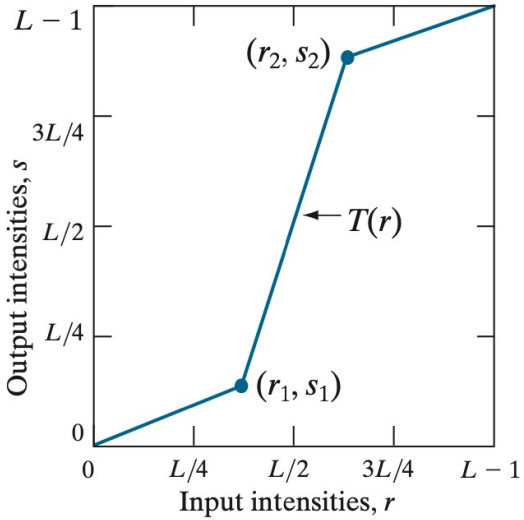
$\gamma = 0.3$

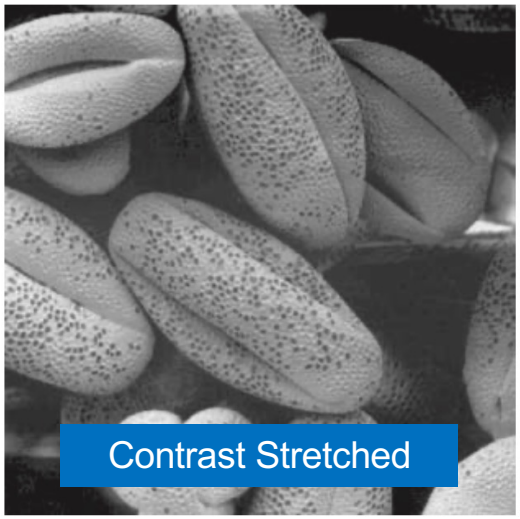
Power-law Contrast Enhancement (2)

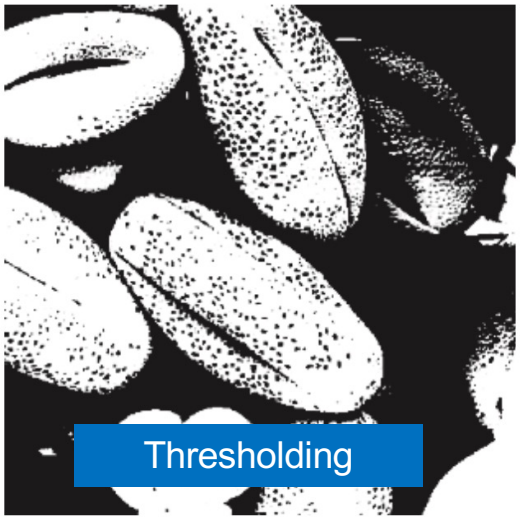


- Original image over exposed
- $\gamma > 1$ darkens the image

Piecewise Linear Transformation (Contrast Stretching)

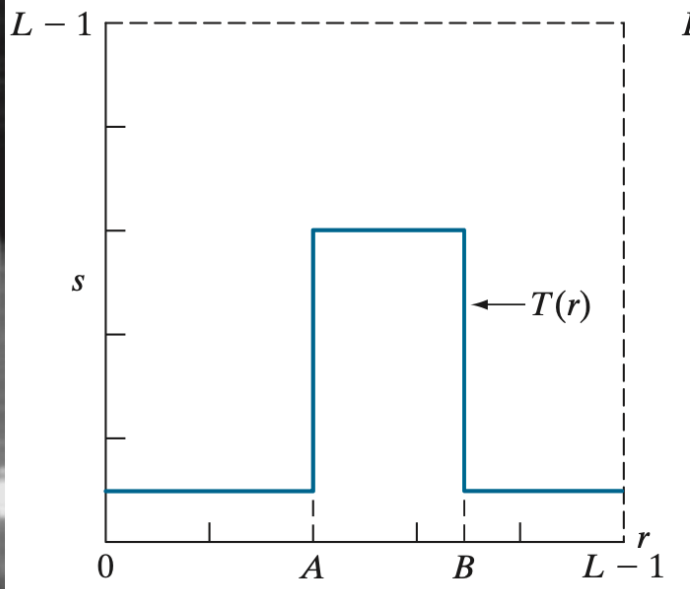


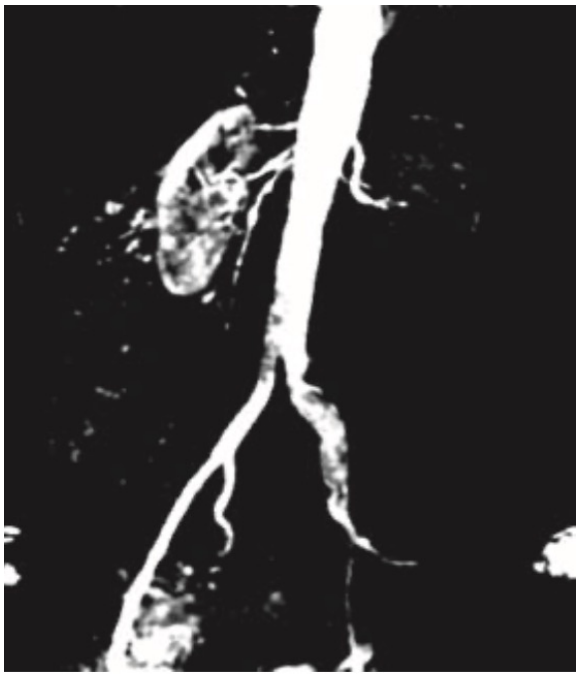




- Transformation function made up of segments of lines
- Gradient of line segments always positive
- Not a mathematical function
- Lighten and darken different grayscale regions
- E.g. low contrast electron microscope image of pollens enlarged by x700
- After contrast stretching using the piecewise linear transformation
- After thresholding only

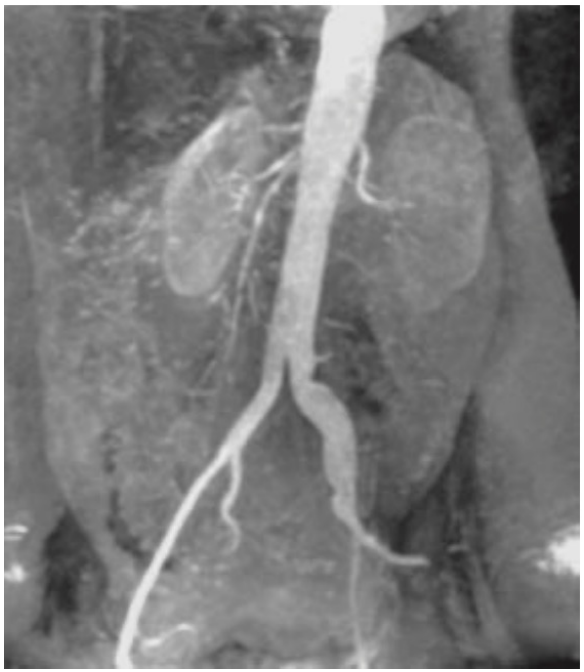
Piecewise Linear Transformation (Intensity-level slicing) (1)

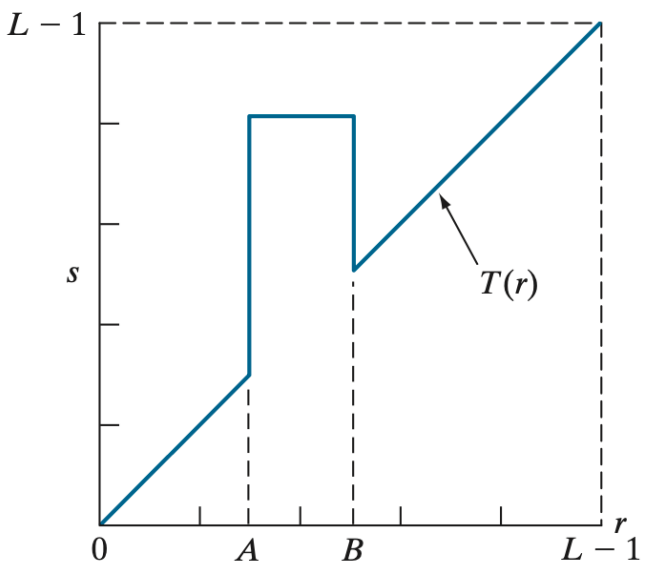


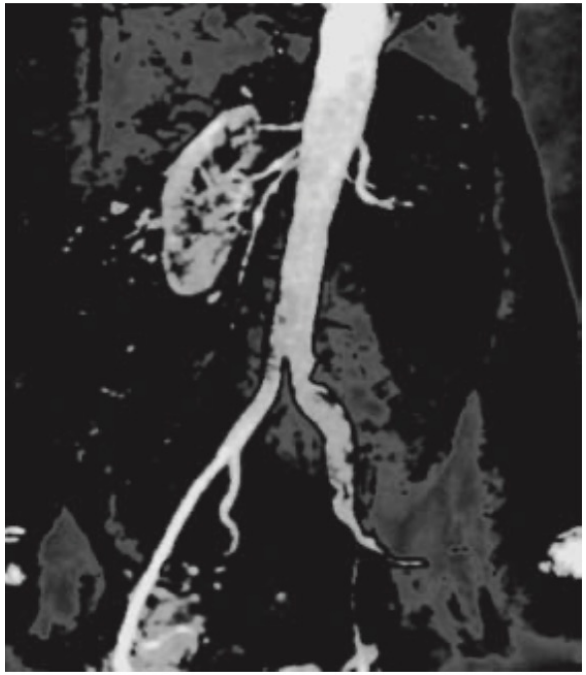


- Similar to thresholding, but use two thresholds instead of one
- Result: highlight region of image with intensity between A and B
- E.g. Aortic angiogram image with interesting feature having intensity with defined region
- Only the arteries and part of heart highlighted

Piecewise Linear Transformation (Intensity-level slicing) (2)

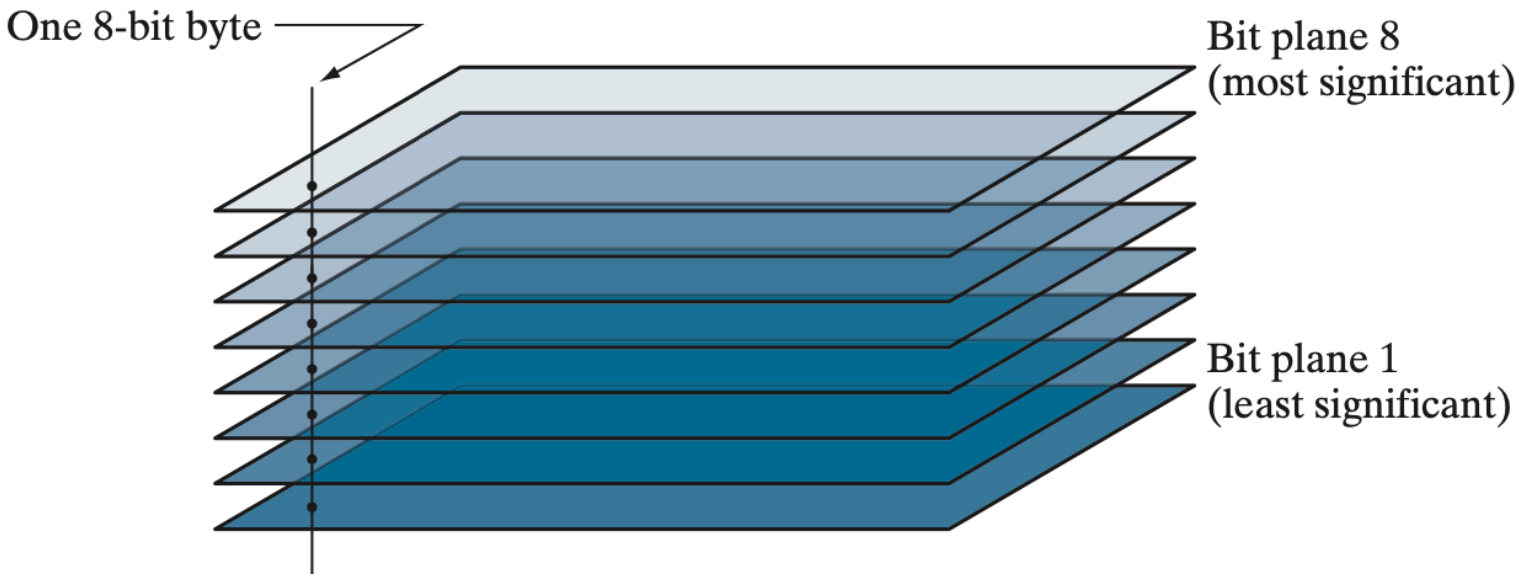






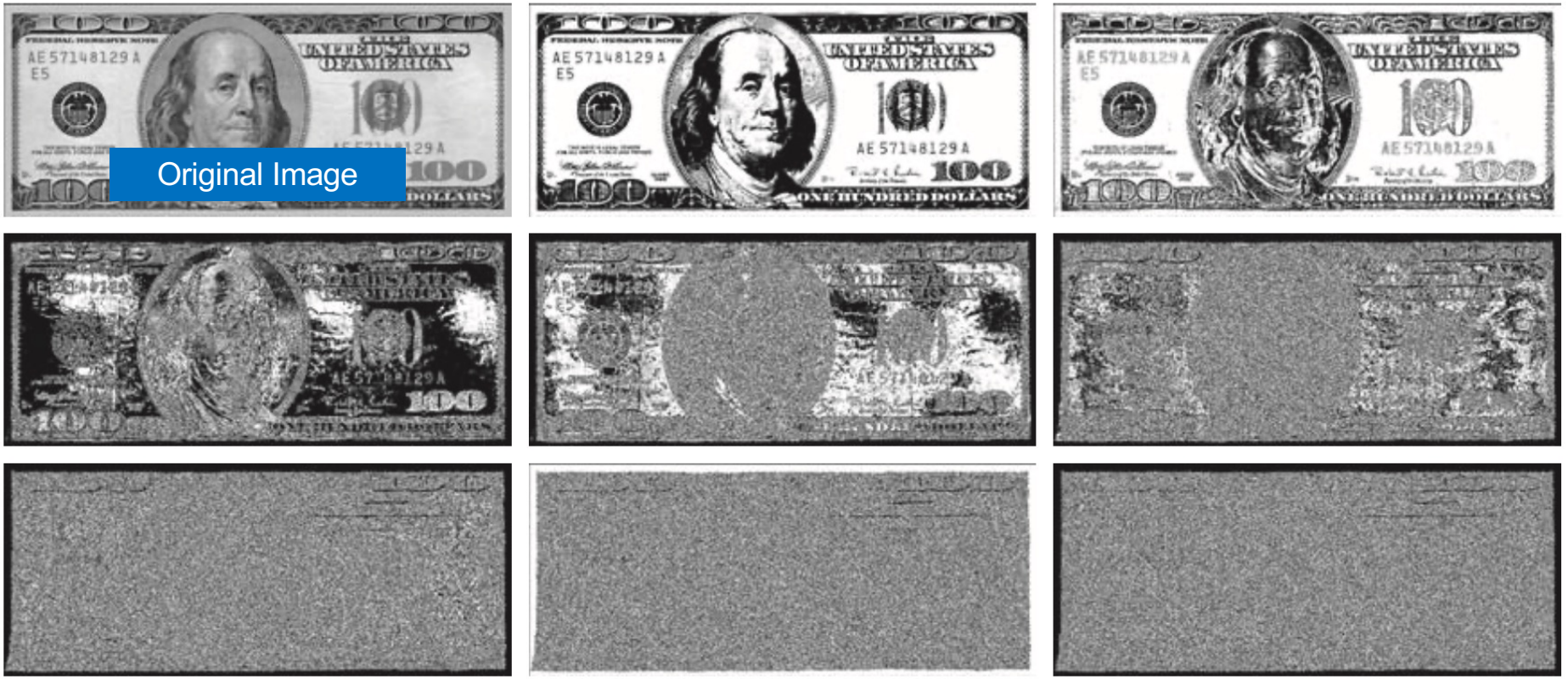
- Similar to the previous, but preserve original image intensity except for the region with intensity between A and B
- Resulting image shows features of other part of body, but still highlighting the region of interest.
- Effectively superimposing the highlighted part onto the original image

Bit-plane slicing



- Intensity is 8-bit number
- Slice each pixel intensity values into images of EACH BIT
- Produces 8 separate images – what for?

Bit-plane slicing of a US bank note (1)



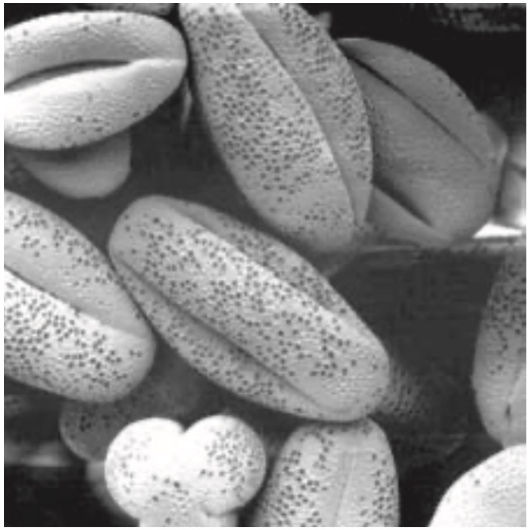
- Slicing original image of bank note shown in top left
- Clearly the most-significant bit image is most important (not surprising)
- Also shows that image of the bottom 4-bits contain little information

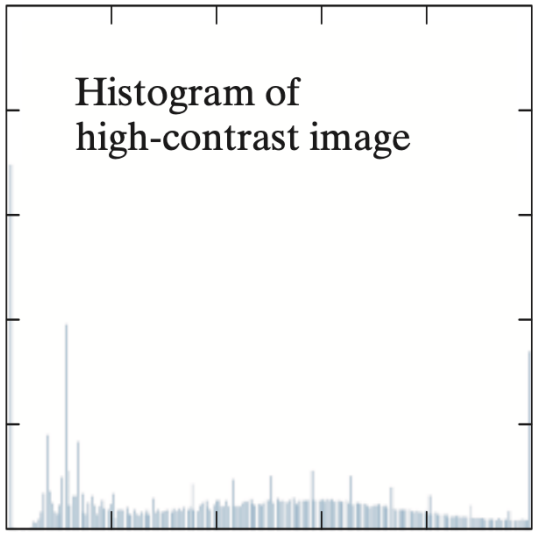
Bit-plane slicing of a US bank note (2)

Original with 8-bit intensity

With only most-significant 4 bits

Formal Definition of Histogram





Let r_k, for $k = 0,1,2,..., L-1$, denote the intensities of an L-level digital image, $f(x, y)$.

The **unnormalized histogram** of f is defined as:

$$h(r_k) = n_k \quad \text{for } k = 0,1,2,\ldots,L-1$$

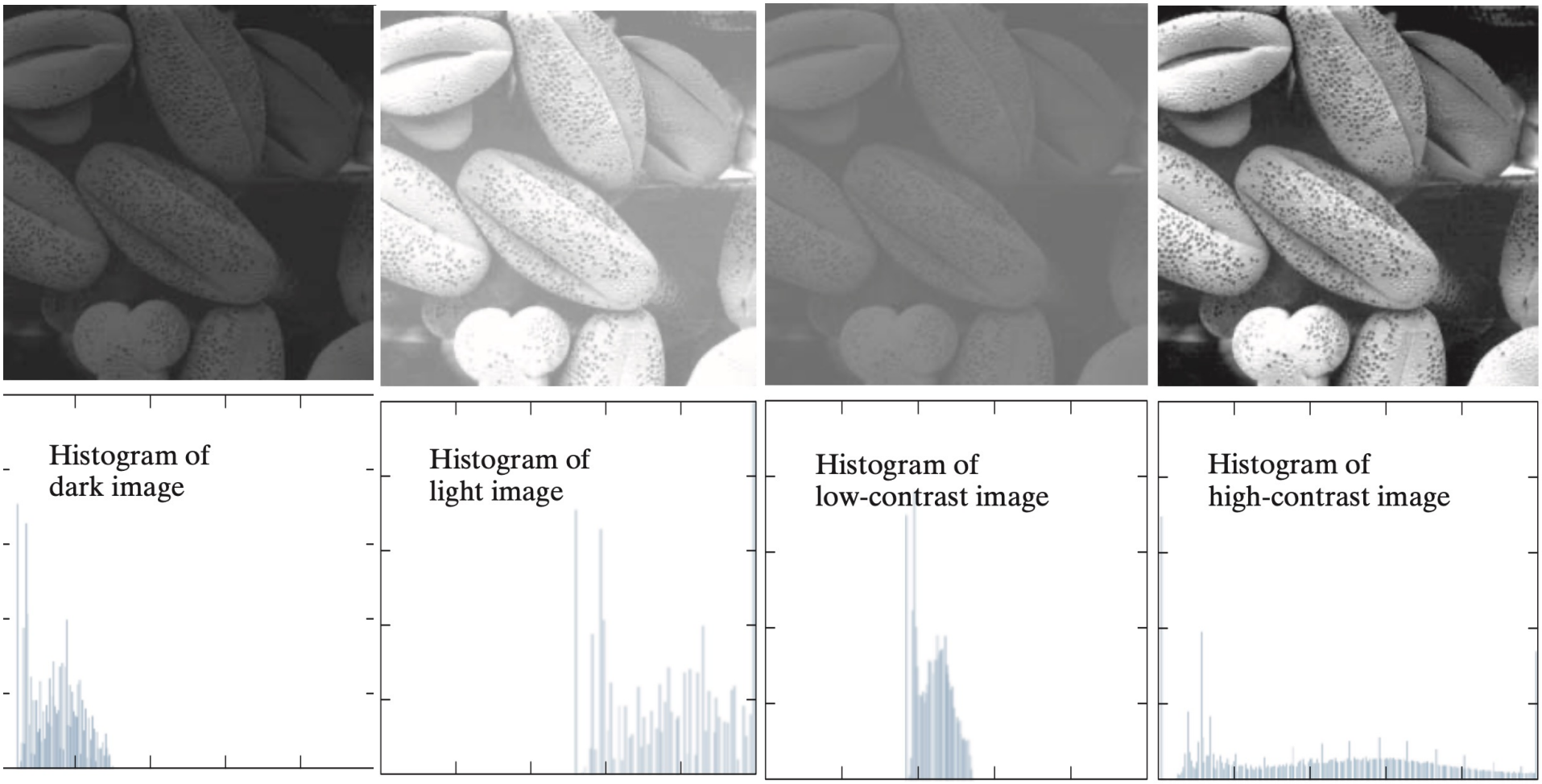
n_k is the number of pixels in f with intensity r_k.

The subdivisions of the intensity scale are called **histogram bins**.

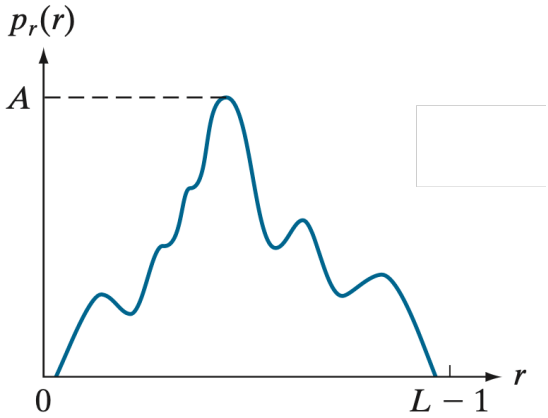
The **normalized histogram** of image f of dimension (MxN)is defined as:

$$p(r_k) = \frac{h(r_k)}{MN} = \frac{n_k}{MN}$$

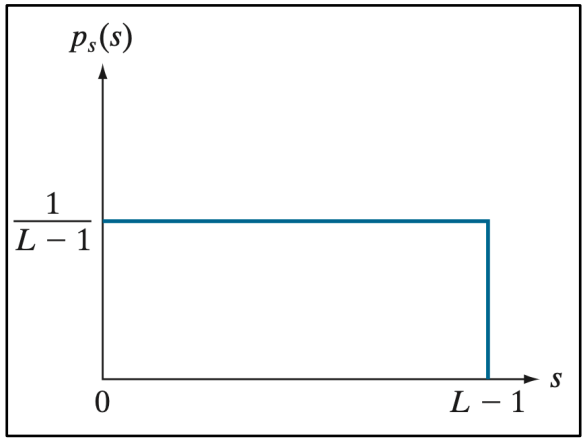
Histogram affects contrast

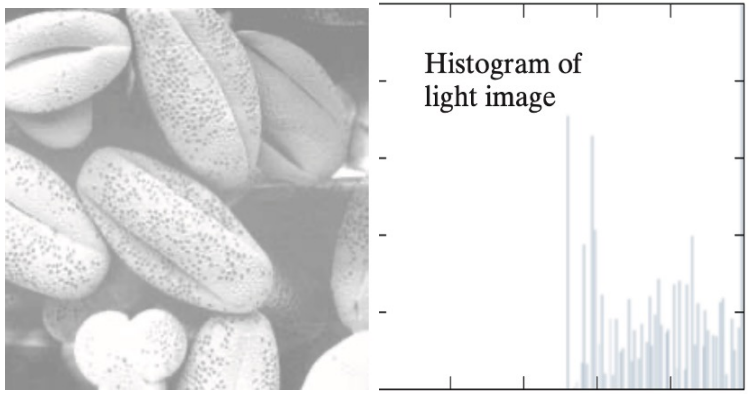


Histogram Equalization

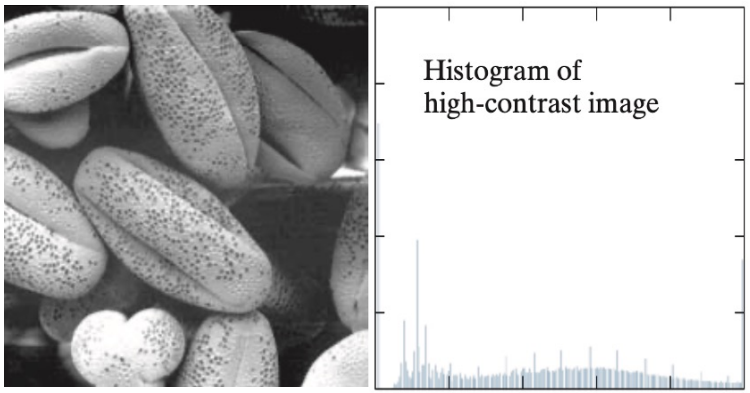


Ideal equalization

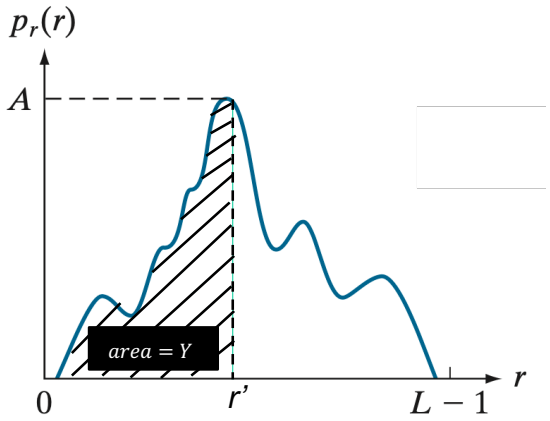




Practical equalization

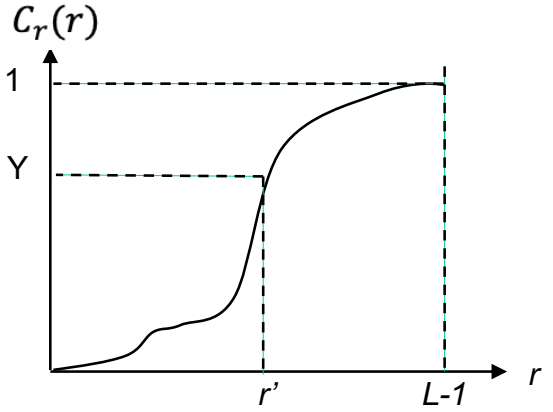


PDF and CDF



Probability Distribution Function PDF

Integrate = area under curve



Type equation here.Cumulative Distribution Function

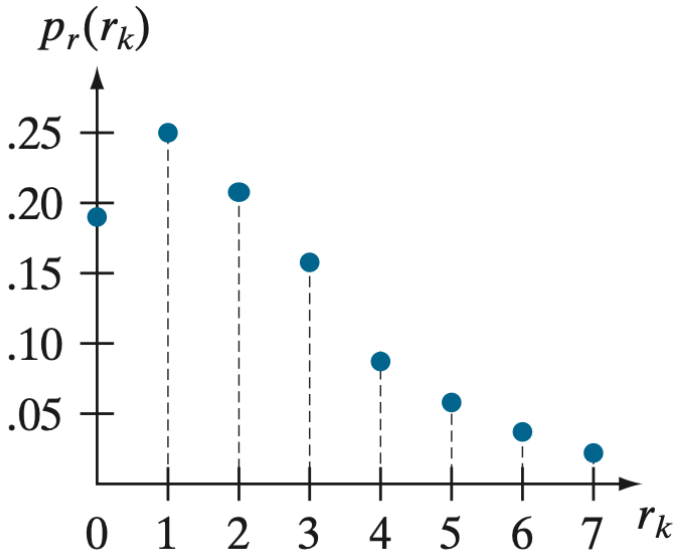
$$C_r(r) = (L-1)\int_0^r p_r(w)dw$$

- Histogram equalization (i.e. flattening of the PDF) can be achieved by using the CDF as the intensity transformation function

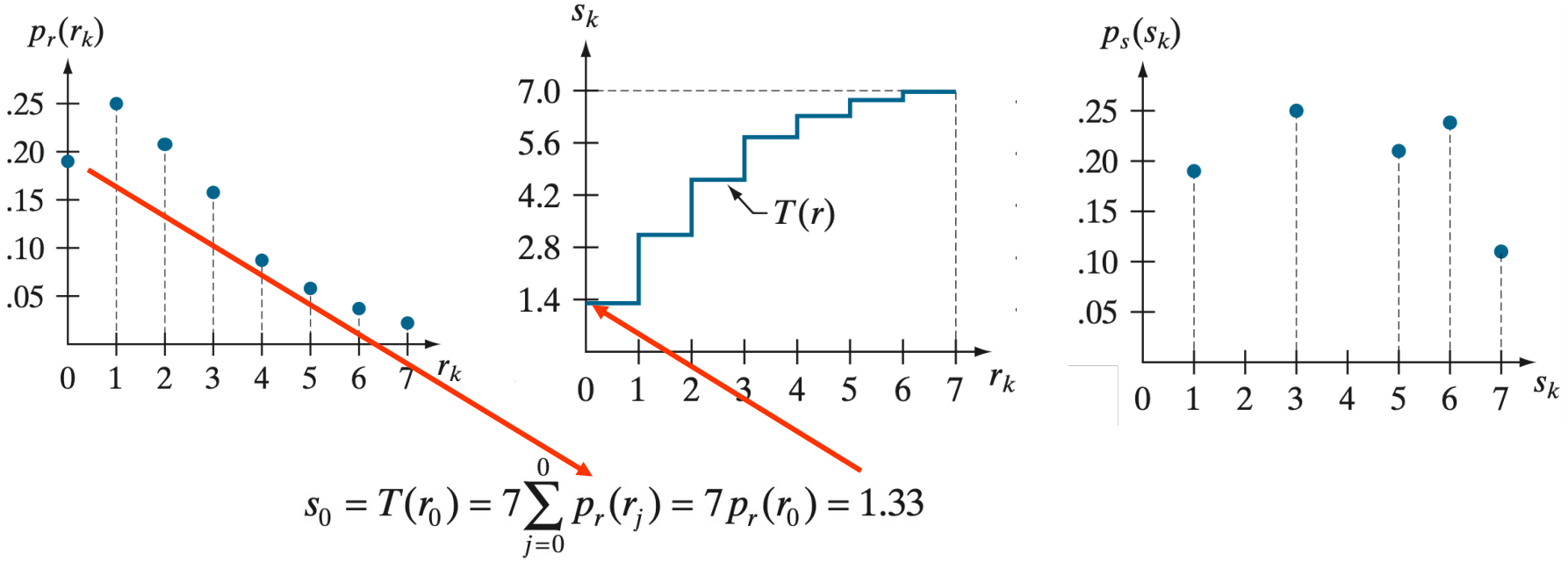
$$s = T(r) = (L-1)\int_0^r p_r(w)dw$$

An example – 3-bit intensity distribution of a 64 x 64 image

r_k	n_k	$p_r(r_k) = n_k / MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02



Compute the CDF and use as intensity transform function

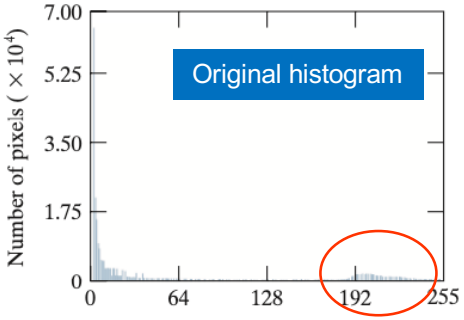


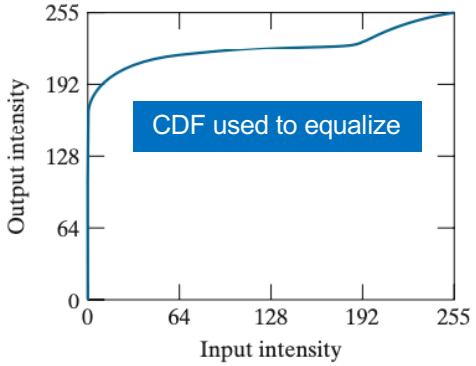
$$s_0 = T(r_0) = 7\sum_{j=0}^{0} p_r(r_j) = 7p_r(r_0) = 1.33$$

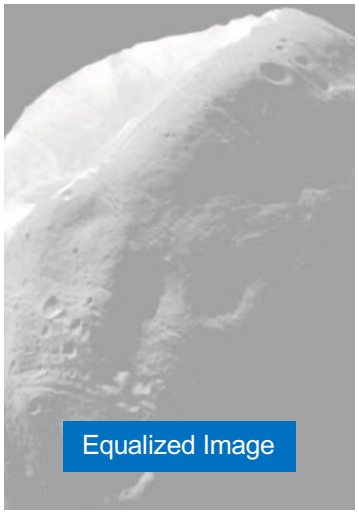
$s_0 = 1.33 \rightarrow 1$	$s_2 = 4.55 \rightarrow 5$	$s_4 = 6.23 \rightarrow 6$	$s_6 = 6.86 \rightarrow 7$
$s_1 = 3.08 \rightarrow 3$	$s_3 = 5.67 \rightarrow 6$	$s_5 = 6.65 \rightarrow 7$	$s_7 = 7.00 \rightarrow 7$

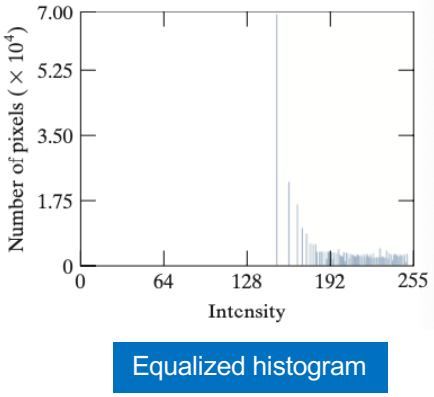
- Note that after equalization, the final histogram have no values at 0 and 2!
- Equalized histogram is approximately flat.

Histogram equalization may not work



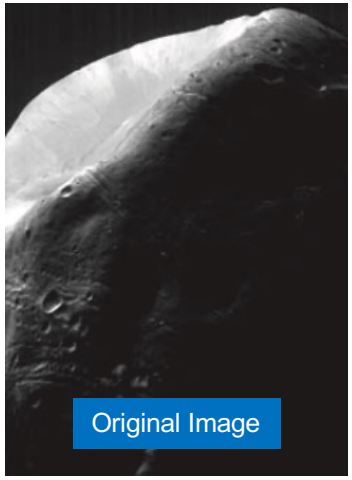


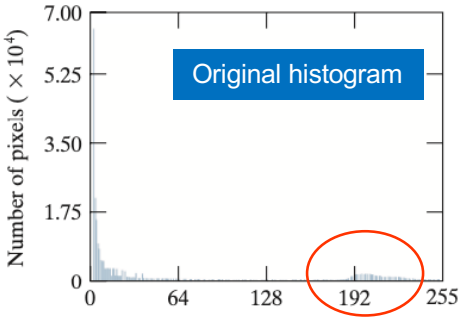


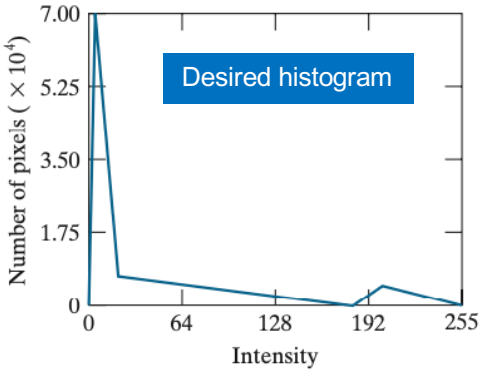


Equalized histogram

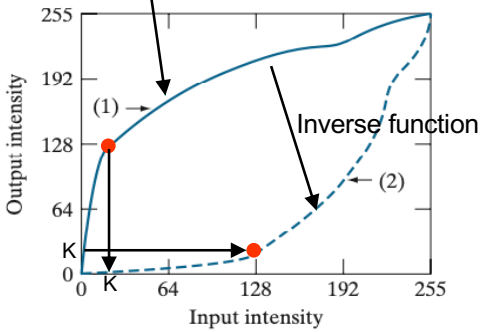
Histogram Matching



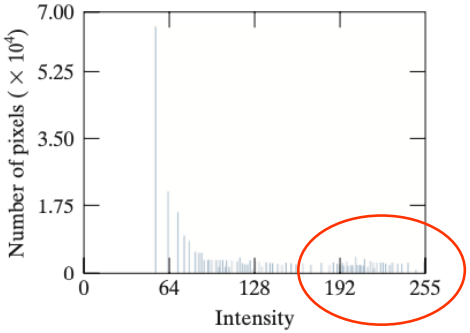




CDF as before



Derived mapping function (dotted line)



matched histogram

Matlab Functions related to this Lecture

imadjust

Adjust image intensity values or colormap

Syntax

```
J = imadjust(I)
J = imadjust(I,[low_in high_in])
J = imadjust(I,[low_in high_in],[low_out high_out])
J = imadjust(I,[low_in high_in],[low_out high_out],gamma)
```

imhist

Histogram of image data

Syntax

```
[counts,binLocations] = imhist(I)
[counts,binLocations] = imhist(I,n)
```

histeq

Enhance contrast using histogram equalization

Syntax

```
J = histeq(I)
J = histeq(I,n)
J = histeq(I,hgram)
```

imhistmatch

Adjust histogram of 2-D image to match histogram of reference image

Syntax

```
J = imhistmatch(I,ref)
J = imhistmatch(I,ref,nbins)
```

stretchlim

Find limits to contrast stretch image

Syntax

```
lowhigh = stretchlim(I)
lowhigh = stretchlim(I,Tol)
```